# Study of infrared emission spectroscopy for the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ and $B^{\prime}{ }^{1}{ }_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ systems of $\mathrm{C}_{2}$ 

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(Received 30 November 2015; accepted 15 January 2016; published online 8 February 2016; publisher error corrected 12 February 2016)


#### Abstract

Thirteen bands for the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ system and eleven bands for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ system of $\mathrm{C}_{2}$ were identified in the Fourier transform infrared emission spectra of hydrocarbon discharges. The $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=4$ and the $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=6,7$, and 8 vibrational levels involved in nine bands were studied for the first time. A direct global analysis with Dunham parameters was carried out satisfactorily for the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ system except for a small perturbation in the $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=6$ level. The calculated rovibrational term energies up to $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=12$ showed that the level crossing between the $B^{1} \Delta_{\mathrm{g}}$ and $d^{3} \Pi_{\mathrm{g}}$ states is responsible for many of the prominent perturbations in the Swan system observed previously. Nineteen forbidden transitions of the $B^{1} \Delta_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}$ transition were identified and the off-diagonal spin-orbit interaction constant $A_{d B}$ between $d^{3} \Pi_{\mathrm{g}}$ and $B^{1} \Delta_{\mathrm{g}}$ was derived as 8.3(1) $\mathrm{cm}^{-1}$. For the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ system, only individual band analyses for each vibrational level in the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state could be done satisfactorily and Dunham parameters obtained from these effective parameters showed that the anharmonic vibrational constant $\omega_{\mathrm{e}} x_{\mathrm{e}}$ is anomalously small (nearly zero). Inspection of the RKR (Rydberg-Klein-Rees) potential curves for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states revealed that an avoided crossing or nearly avoided crossing may occur around $30000 \mathrm{~cm}^{-1}$, which is responsible for the anomalous molecular constants in these two states. © 2016 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4940907]


## I. INTRODUCTION

Recently, progress on the perturbation analysis of $\mathrm{C}_{2}$ spectra has appeared in several studies for the tripletquintet $\left(d^{3} \Pi_{\mathrm{g}}-l^{5} \Pi_{\mathrm{g}}\right)$ interaction, ${ }^{1}$ triplet-triplet $\left(c^{3} \Sigma_{\mathrm{u}}{ }^{+}-a^{3} \Pi_{\mathrm{u}}\right)$ interaction, ${ }^{2}$ and singlet-triplet $\left(X^{1} \Sigma_{\mathrm{g}}{ }^{+}-b^{3} \Sigma_{\mathrm{g}}{ }^{-}\right)$interaction. ${ }^{3}$ These interactions cause prominent perturbations, unusual forbidden triplet-quintet intersystem transitions, ${ }^{1}$ forbidden singlet-triplet intersystem transitions, ${ }^{3}$ and the observation of a quintet-quintet band. ${ }^{4}$ These studies aroused further interest in other $\mathrm{C}_{2}$ spectra for the many low-lying electronic states (Fig. 1) of this fundamental molecule.

The $B^{1} \Delta_{\mathrm{g}}$ and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$states of $\mathrm{C}_{2}$ were first predicted by Phillips ${ }^{5}$ to explain the perturbations in the $\mathrm{v}=4$ and 5 vibrational levels of the $d^{3} \Pi_{\mathrm{g}}$ state. In 1988, the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ band systems were discovered by Douay et al., ${ }^{6}$ who observed eight bands of the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ transition with v up to 5 for the $B^{1} \Delta_{\mathrm{g}}$ state and six bands of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ transition with v up to 3 for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state. The molecular constants obtained for $B^{1} \Delta_{\mathrm{g}}$ were very well behaved, but those for $B^{\prime \prime} \Sigma_{\mathrm{g}}{ }^{+}$were anomalous, for example, with a small anharmonic vibrational constant $\omega_{\mathrm{e}} x_{\mathrm{e}}$. It was suspected that interactions with the $X^{1} \Sigma_{g}{ }^{+}$state were responsible. ${ }^{6}$ Recently,

[^0]$a b$ initio potential energy curves were calculated for the low-lying singlet states of $\mathrm{C}_{2}$ and the anomalous value of $\omega_{\mathrm{e}} x_{\mathrm{e}}$ was reproduced by the theoretical work. ${ }^{7,8}$ Other band systems related to the $B^{1} \Delta_{\mathrm{g}}$ and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$states were also observed, such as the $1^{1} \Delta_{\mathrm{u}}-B^{1} \Delta_{\mathrm{g}}$ transition by REMPI spectroscopy ${ }^{9}$ and the $D^{1} \Sigma_{\mathrm{u}}{ }^{+}-B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$transition by laser induced fluorescence of the photodissociation fragments ${ }^{10,11}$ and in solid Ne. ${ }^{12}$

In the study of the Swan system $\left(d^{3} \Pi_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}\right)$ of $\mathrm{C}_{2}$, many prominent perturbations in the $d^{3} \Pi_{\mathrm{g}}$ state could be explained by the interactions at the $d^{3} \Pi_{\mathrm{g}}-b^{3} \Sigma_{\mathrm{g}}{ }^{-}$crossing points, but many unidentified perturbations were suspected due to interactions with the vibrationally excited levels of the $B^{1} \Delta_{\mathrm{g}}$ and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$ states. ${ }^{13}$ In particular, a puzzling perturbation, which cannot be due to a $b^{3} \Sigma_{\mathrm{g}}{ }^{-}$level, was revealed to cross between the $J=9$ and $J=10$ levels of the $F_{2}$ spin component for the $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$ vibrational level with a $1 \mathrm{~cm}^{-1}$ perturbation shift to higher wavenumbers and weak extra lines on the lower wavenumber side. ${ }^{13}$

In the work presented here, we identified additional bands with higher vibrational levels for the $B^{1} \Delta_{\mathrm{g}}$ and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$states. As a result, the $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=9$ level is found to be responsible for perturbing the $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$ level, and the extra lines observed previously ${ }^{13}$ were just the forbidden transitions with considerable intensity borrowed from the allowed transitions. For the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state, the anomalous anharmonic vibrational constant $\omega_{\mathrm{e}} x_{\mathrm{e}}$ is likely caused by an avoided crossing between the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$potential curves.


FIG. 1. Low-lying electronic states of $C_{2}$ below $30000 \mathrm{~cm}^{-1}$ and observed transition systems (vertical solid lines) within the same multiplicity. Intersystem forbidden transitions (dashed lines) were observed recently for $1^{5} \Pi_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}$ (Ref. 1), $X^{1} \Sigma_{\mathrm{g}}{ }^{+}-a^{3} \Pi_{\mathrm{u}}$ and $A^{1} \Pi_{\mathrm{u}}-b^{3} \Sigma_{\mathrm{g}}{ }^{-}$(Ref. 3), and $B^{1} \Delta_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}$ in the present work.

## II. SPECTRAL ASSIGNMENT

During our previous study, ${ }^{3}$ we noticed some new bands around $3500 \mathrm{~cm}^{-1}$ in the Fourier transform infrared (FTIR) emission spectrum of $\mathrm{C}_{2}$ observed by a positive column discharge in the $\mathrm{CH}_{4}$ and He mixture with a spectral resolution of $0.02 \mathrm{~cm}^{-1} .{ }^{14}$ Using the molecular constants of Douay et al., ${ }^{6}$ we could assign these bands easily to $\mathrm{v}=0-2,1-3,2-4$, and 3-5 of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ system. The variation of the line intensity for these $\Delta \mathrm{v}=-2$ bands indicated that the $\mathrm{v}=4-6$ band might be observable, but the 5-7 band might be too weak to be seen. Using the equilibrium molecular constants of Douay et al. derived for $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$, ${ }^{6}$ we estimated the band position for the $\mathrm{v}=4-6$ band and were able to assign the transition around $3470 \mathrm{~cm}^{-1}$, about $5.8 \mathrm{~cm}^{-1}$ lower than the predicted. A portion of the spectrum mainly showing the $Q$-branch of the $v=4-6$ band is shown in Fig. 2.

TABLE I. Assigned $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ and $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ bands for $\mathrm{C}_{2}$.

|  | $\Delta \mathrm{v}$ |  |  | $\mathrm{v}^{\prime}-\mathrm{v}^{\prime \prime}$ | $J_{\text {max }}$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ | -2 | 0-2(30) | 1-3(32) | 2-4(30) | 3-5(28) | 4-6(26) |  |
|  | -1 | $0-1(37)^{\text {a }}$ | $1-2(41)^{\text {a }}$ | 2-3(40) | 3-4(34) | 4-5(26) |  |
|  | 0 | $0-0(41)^{\text {a }}$ | 1-1(35) | 2-2(26) |  |  |  |
|  | 1 | $1-0(40)^{\text {a }}$ | $2-1(38)^{\text {a }}$ | $3-2(34)^{\text {a }}$ | 4-3(28) |  |  |
| $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ |  | $0-1(26)^{\text {a }}$ |  |  | 5-5(24) |  | 8-5(24) |
|  | 0 | $0-0(40)^{\text {a }}$ | 1-1(27) | 4-4(15) |  |  |  |
|  | 1 | $1-0(46)^{\text {a }}$ | $2-1(37)^{\text {a }}$ | $3-2(34)^{\text {a }}$ |  |  |  |
|  | 2 | 2-0(40) | $3-1(39)^{\text {a }}$ | $4-2(36)^{a}$ | 5-3(31) | 6-4(26) |  |
|  | 3 | 3-0(34) | 4-1(33) | $5-2(33)^{a}$ | 6-3(36) | 7-4(34) |  |
|  | 4 | 6-2(24) | 7-3(33) |  |  |  |  |

${ }^{\text {a }}$ Bands assigned previously by Douay et al. ${ }^{6}$ were extended to higher $J$ values.

With the newly determined molecular constants for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=4$ level, as shown later in Table VII, we extended the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ assignment to the higher wavenumber region by using two previously observed spectra by Douay et al. ${ }^{15}$ and could assign the $v=2-3,3-4$, and $4-5$ bands around $5000 \mathrm{~cm}^{-1}$, the $\mathrm{v}=1-1$, and 2-2 bands around $7000 \mathrm{~cm}^{-1}$, and the $\mathrm{v}=4-3$ band around $8000 \mathrm{~cm}^{-1}$, as summarized in Table I. Among these assigned bands, the wavenumbers for three bands with $\mathrm{v}=4$ for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state are listed in Table II.

For the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ system, seven bands up to $\mathrm{v}=5$ for the $B^{1} \Delta_{\mathrm{g}}$ state were assigned in this work using the molecular constants of Douay et al. ${ }^{6}$ In addition, six bands with $\mathrm{v}=6,7$, and 8 for the $B^{1} \Delta_{\mathrm{g}}$ state were assigned using the equilibrium molecular constants of Douay et al. ${ }^{6}$ and the transition wavenumbers are listed in Table III. All the assigned $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ bands are summarized in Table I with vibrational quantum numbers.

## III. ANALYSIS

For the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ system, we first analyzed the individual vibrational levels up to $\mathrm{v}=8$ of the upper $B^{1} \Delta_{\mathrm{g}}$ state by fixing Dunham parameters of the lower $A^{1} \Pi_{u}$ state to those of our previous work, ${ }^{3}$ as shown in Table IV. These effective molecular constants are compared with those of Douay et al. Then, a direct global analysis using Dunham parameters for


FIG. 2. A portion of the spectrum showing the Q - and P-branches of $\mathrm{v}=4-6$ and $\mathrm{v}=3-5$ bands for the $B^{\prime \prime} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ system. The lines marked with belong to the $\mathrm{v}=0-0$ band for the $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ system. The lines marked with $\diamond$ belong to the $b^{3} \Sigma_{\mathrm{g}}{ }^{-}-a^{3} \Pi_{\mathrm{u}}$ system as assigned in Ref. 3 .

TABLE II. $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ transitions with $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=4\left(\mathrm{in} \mathrm{cm}^{-1}\right)$. Observed minus calculated in units of $10^{-4} \mathrm{~cm}^{-1}$.

| 4-3 band |
| :--- |


| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{P}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ | $J$ | $\mathrm{Q}_{\mathrm{ef}}$ | $\mathrm{O}-\mathrm{C}$ |
| :--- | :---: | ---: | :---: | ---: | :---: | :---: | :---: |
| 1 | 7935.7065 | -7 | 7927.0852 | -23 |  |  |  |
| 3 | 7940.2439 | $-82^{\mathrm{a}}$ | 7920.1429 | 12 | 2 | 7929.4766 | -31 |
| 5 | 7943.8346 | 8 | 7912.2365 | -10 | 4 | 7927.8044 | $76^{\mathrm{a}}$ |
| 7 | 7946.4517 | 13 |  |  | 6 | 7925.1512 | -6 |
| 9 | 7948.0995 | -4 | 7893.5547 | -26 | 8 | 7921.5433 | -6 |
| 11 | 7948.7802 | 3 | 7882.7847 | 24 | 10 | 7916.9723 | -1 |
| 13 | 7948.4856 | 26 | 7871.0503 | -8 | 12 | 7911.4329 | -35 |
| 15 | 7947.2195 | -31 | 7858.3675 | 34 | 14 | 7904.9334 | -13 |
| 17 | 7944.9710 | $-102^{\mathrm{a}}$ | 7844.7187 | -28 | 16 | 7897.4643 | -22 |
| 19 | 7941.7607 | -13 | 7830.1269 | 30 | 18 | 7889.0281 | -29 |
| 21 | 7937.5740 | $102^{\mathrm{a}}$ | 7814.5686 | -33 | 20 | 7879.6250 | -28 |
| 23 |  |  | 7798.0677 | 9 | 22 |  |  |
| 25 |  |  |  |  | 24 | 7857.9194 | 9 |
| 27 |  |  |  |  | 26 | 7845.6143 | $0^{\mathrm{b}}$ |
| 29 |  |  |  |  | 28 | 7832.3132 | $-332^{\mathrm{a}}$ |


| 4-5 band |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | O-C | $\mathrm{P}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ | $\boldsymbol{J}$ | $\mathrm{Q}_{\mathrm{ef}}$ | O-C |
| 3 |  |  | 4922.5533 | $56^{\mathrm{b}}$ | 2 | 4931.6808 | 35 |
| 5 | 4946.8684 | 34 | 4915.2708 | 22 | 4 | 4930.4824 | 17 |
| 7 | 4950.3749 | $-97^{\mathrm{a}}$ | 4907.3122 | 22 | 6 | 4928.6011 | 12 |
| 9 | 4953.2198 | $48^{\mathrm{b}}$ | 4898.6774 | $49^{\mathrm{b}}$ | 8 | 4926.0360 | 19 |
| 11 | 4955.3594 | $53^{\mathrm{b}}$ | 4889.3702 | $137^{\mathrm{a}}$ | 10 | 4922.7859 | 32 |
| 13 | 4956.8010 | 15 | 4879.3641 | 17 | 12 | 4918.8476 | 28 |
| 15 | 4957.5493 | 0 | 4868.6933 | 25 | 14 | 4914.2211 | 16 |
| 17 | 4957.6022 | 7 | 4857.3369 | $-50^{\mathrm{b}}$ | 16 | 4908.9100 | 39 |
| 19 | 4956.9541 | -4 | 4845.3193 | 29 | 18 | 4902.9024 | -13 |
| 21 | 4955.6159 | $87^{\mathrm{a}}$ | 4832.6156 | 3 | 20 | 4896.2108 | -15 |
| 23 | 4953.5596 | 6 | 4819.2405 | 5 | 22 | 4888.8309 | -9 |
| 25 | 4950.8076 | -25 | 4805.1938 | 14 | 24 | 4880.7637 | 6 |
| 27 |  |  | 4790.4844 | $91^{\mathrm{a}}$ | 26 | 4872.0100 | 24 |


| 4-6 band |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | O-C | $\mathrm{P}_{\mathrm{ee}}$ | O-C | $\boldsymbol{J}$ | $\mathrm{Q}_{\mathrm{ef}}$ | O-C |
| 1 | 3475.5108 | -12 |  |  |  |  |  |
| 3 | 3480.5759 | -34 | 3460.4609 | $-80^{\mathrm{a}}$ | 2 | 3469.4865 | $-70^{\mathrm{a}}$ |
| 5 | 3485.0968 | $-47^{\mathrm{b}}$ | 3453.4982 | $-69^{\mathrm{b}}$ | 4 | 3468.5426 | 5 |
| 7 | 3489.0763 | -2 | 3445.9978 | -41 | 6 | 3467.0421 | $-45^{\mathrm{b}}$ |
| 9 | 3492.4971 | $-55^{\mathrm{b}}$ | 3437.9589 | -11 | 8 | 3465.0058 | -6 |
| 11 | 3495.3701 | $-73^{\mathrm{b}}$ | 3429.3734 | $-64^{\mathrm{b}}$ | 10 | 3462.4156 | $-52^{\mathrm{b}}$ |
| 13 | 3497.6919 | $-70^{\mathrm{b}}$ | 3420.2557 | $-61^{\mathrm{b}}$ | 12 | 3459.2837 | $-52^{\mathrm{b}}$ |
| 15 | 3499.4558 | $-93^{\mathrm{a}}$ | 3410.5993 | $-72^{\mathrm{b}}$ | 14 | 3455.5970 | $-129^{\mathrm{a}}$ |
| 17 | 3500.6631 | $-109^{\mathrm{a}}$ | 3400.4042 | $-102^{\mathrm{a}}$ | 16 | 3451.3876 | $45^{\mathrm{b}}$ |
| 19 |  |  | 3389.6620 | $-242^{\mathrm{a}}$ | 18 | 3446.6157 | $78^{\mathrm{a}}$ |
| 21 |  |  |  |  | 20 | 3441.2916 | $76^{\mathrm{b}}$ |
| 23 |  |  |  |  | 22 | 3435.4147 | 30 |
| 25 |  |  |  |  | 24 | 3428.9931 | 12 |
| 27 |  |  |  |  | 26 | 3422.0235 | -26 |

[^1]TABLE III. $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ transitions with $\mathrm{v}=6,7,8$ for $B^{1} \Delta_{\mathrm{g}}$ (in $\mathrm{cm}^{-1}$ ). Observed minus calculated in units of $10^{-4} \mathrm{~cm}^{-1}$.

| 6-2 band |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | Qef | O-C | $\mathrm{P}_{\mathrm{ff}}$ | O-C |
| 2 | 8419.0748 | $-89^{\text {a }}$ |  |  |  |  |
| 4 | 8421.4083 | -45 | 8407.8806 | 25 |  |  |
| 6 | 8421.9700 | -68 | 8403.0299 | -26 |  |  |
| 8 | 8420.7817 | $70^{\text {a }}$ | 8396.4203 | -48 | 8374.7674 | $-102^{\text {b }}$ |
| 10 | 8417.8035 | -17 | 8388.0514 | -47 | 8360.9955 | $-104{ }^{\text {b }}$ |
| 12 | 8413.0642 | -33 | 8377.9212 | -43 | 8345.4780 | -10 |
| 14 | 8406.5632 | 28 | 8366.0323 | -14 | 8328.1907 | $-75^{\text {a }}$ |
| 16 | 8398.2903 | $72^{\text {a }}$ |  |  |  |  |
| 18 | 8388.2335 | -11 | 8336.9654 | -20 |  |  |
| 20 | 8376.4130 | -10 | 8319.7947 | 10 |  |  |
| 22 |  |  | 8300.8606 | 6 |  |  |
| 24 | 8347.4548 | 15 | 8280.1677 | 7 |  |  |
| $J$ | $\mathrm{R}_{\text {ee }}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\text {ee }}$ | O-C |
| 1 | 8417.2533 | -47 |  |  |  |  |
| 3 | 8420.4716 | 5 | 8409.6491 | 67 |  |  |
| 5 | 8421.9140 | $-73^{\text {a }}$ | 8405.6904 | $91^{\text {b }}$ |  |  |
| 7 | 8421.6097 | 23 | 8399.9599 | 1 |  |  |
| 9 | 8419.5304 | 22 | 8392.4759 | -22 | 8368.1259 | -26 |
| 11 | 8415.6777 | -51 | 8383.2315 | -47 | 8353.4809 | -62 |
| 13 | 8410.0688 | -12 | 8372.2278 | -68 | 8337.0880 | -45 |
| 15 | 8402.6879 | -11 |  |  | 8318.9449 | -16 |
| 17 | 8393.5351 | -36 | 8344.9445 | $-81^{\text {a }}$ | 8299.0568 | 65 |
| 19 | 8382.6403 | $221^{\text {b,c }}$ | 8328.6993 | $264{ }^{\text {b,c }}$ |  |  |
| 21 | 8369.9332 | $65^{\text {b,c }}$ |  |  |  |  |
| 23 | 8355.4651 | 18 | 8290.8434 | 54 |  |  |
| 25 | 8339.2296 | 24 |  |  |  |  |
|  |  |  | 6-3 band |  |  |  |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | $\mathrm{Q}_{\text {ef }}$ | O-C | $\mathrm{P}_{\mathrm{ff}}$ | O-C |
| 2 |  |  | 6875.5049 | -18 |  |  |
| 4 |  |  | 6872.6615 | -28 |  |  |
| 6 | 6887.1473 | 53 | 6868.1989 | 12 | 6851.9451 | $-126^{\text {b }}$ |
| 8 | 6886.4590 | 24 | 6862.1102 | 31 | 6840.4594 | -2 |
| 10 | 6884.1376 | -43 | 6854.3921 | -6 | 6827.3463 | 37 |
| 12 | 6880.1815 | $-152^{\text {b }}$ | 6845.0543 | -4 | 6812.6119 | 37 |
| 14 | 6874.6135 | -67 | 6834.0977 | 42 | 6796.2606 | 26 |
| 16 | 6867.4147 | 32 | 6821.5088 | -5 | 6778.2945 | 10 |
| 18 | 6858.5884 | $188^{\text {b }}$ | 6807.3029 | 4 | 6758.7167 | 3 |
| 20 | 6848.0939 | 0 | 6791.4734 | -2 | 6737.5210 | $-73^{\text {a }}$ |
| 22 | 6835.9854 | 19 | 6774.0225 | -6 | 6714.7292 | -18 |
| 24 | 6822.2373 | -5 | 6754.9347 | $-168^{\text {b }}$ | 6690.3566 | $304{ }^{\text {b }}$ |
| 26 |  |  | 6734.2685 | $91^{\text {b }}$ | 6664.3182 | 23 |
| 28 | 6789.8402 | 25 | 6711.9436 | -38 | 6636.7046 | 29 |
| 30 |  |  | 6688.0155 | -8 | 6607.4828 | -30 |
| 34 |  |  | 6635.2968 | -28 |  |  |
| 36 |  |  | 6606.5182 | 26 |  |  |
| $J$ | $\mathrm{R}_{\text {ee }}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\text {ee }}$ | O-C |
| 1 | 6881.7340 | -2 |  |  |  |  |
| 3 | 6885.1194 | -1 | 6874.2882 | -26 |  |  |
| 5 | 6886.8806 | 9 | 6870.6442 | 45 |  |  |
| 7 | 6887.0170 | 34 | 6865.3810 | $150^{\text {b }}$ |  |  |
| 9 | 6885.5328 | $127{ }^{\text {b }}$ | 6858.4781 | $81^{\text {a }}$ |  |  |
| 11 | 6882.4059 | $76^{\text {a }}$ | 6849.9518 | 1 |  |  |
| 13 |  |  | 6839.8105 | -10 |  |  |
| 15 | 6871.2693 | 39 | 6828.0553 | 56 | 6787.5332 | $103{ }^{\text {b }}$ |

TABLE III.(Continued.)

|  |  | 6-3 band |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\mathrm{ee}}$ | O-C |  |  |
| 17 | 6863.2533 | 7 | 6814.6385 | $-280^{\mathrm{b}}$ | 6768.7573 | $-70^{\mathrm{a}}$ |  |  |
| 19 | 6853.6072 | -6 | 6799.6637 | 12 | 6748.3950 | -3 |  |  |
| 21 | 6842.3405 | $104^{\mathrm{b}}$ |  |  | 6726.4093 | $-84^{\mathrm{a}}$ |  |  |
| 23 |  |  | 6764.7885 | -51 | 6702.8448 | $116^{\mathrm{b}}$ |  |  |
| 25 | 6814.8653 | $-80^{\mathrm{a}}$ | 6744.9513 | $215^{\mathrm{b}}$ | 6677.6380 | -55 |  |  |
| 27 |  |  | 6723.4323 | $-148^{\mathrm{b}}$ | 6650.8515 | 11 |  |  |
| 29 |  |  | 6700.3409 | -54 | 6622.4574 | 14 |  |  |
| 31 |  |  | 6675.6290 | 10 |  |  |  |  |


| 6-4 band |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | Qef | O-C | $\mathrm{P}_{\mathrm{ff}}$ | O-C |
| 2 | 5372.5102 | -64 | 5364.3910 | -35 |  |  |
| 4 | 5375.3340 | 47 | 5361.7978 | 32 |  |  |
| 6 | 5376.6647 | $114^{\text {b }}$ | 5357.7117 | 27 |  |  |
| 8 | 5376.4908 | 32 | 5352.1454 | 74 |  |  |
| 10 | 5374.8346 | 36 | 5345.0915 | $96^{\text {b }}$ |  |  |
| 12 | 5371.6865 | 37 | 5336.5430 | 22 |  |  |
| 14 | 5367.0457 | 38 | 5326.5176 | 25 |  |  |
| 18 |  |  | 5302.0113 | -4 |  |  |
| 20 |  |  | 5287.5392 | 43 |  |  |
| 24 |  |  | 5254.1273 | -69 |  |  |
|  |  |  | 5235.2164 | 49 |  |  |
| $J$ | $\mathrm{R}_{\text {ee }}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\text {ee }}$ | O-C |
| 3 | 5374.1167 | 54 | 5363.2867 | 42 |  |  |
| 5 |  |  | 5359.9451 | 20 |  |  |
| 7 | 5376.7696 | 23 | 5355.1249 | 51 |  |  |
| 9 | 5375.8719 | $92^{\text {b }}$ |  |  |  |  |
| 11 |  |  | 5341.0138 | $-80^{\text {b }}$ |  |  |
| 13 | 5369.5796 | -36 |  |  |  |  |
| 15 |  |  | 5320.9910 | 2 | 5280.4659 | 19 |
| 17 |  |  | 5308.7474 | -39 |  |  |
| 21 |  |  | 5279.8274 | 6 |  |  |
| 23 |  |  | 5263.1381 | -48 |  |  |
| 25 |  |  | 5244.9620 | $-168^{\text {b }}$ |  |  |


| 7-3 band |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | Qef | O-C | $\mathrm{P}_{\mathrm{ff}}$ | O-C |
| 2 | 8131.7220 | $-109{ }^{\text {b }}$ | 8123.7209 | $77^{\text {a }}$ |  |  |
| 4 | 8133.9857 | $-101^{\text {b }}$ | 8120.6340 | 22 |  |  |
| 6 | 8134.4811 | $-138{ }^{\text {b }}$ | 8115.7894 | -2 |  |  |
| 8 | 8133.2259 | -32 | 8109.1847 | -21 |  |  |
| 10 | 8130.1935 | -38 | 8100.8219 | -18 | 8074.1224 | $74^{\text {a }}$ |
| 12 | 8125.3976 | -11 | 8090.7020 | 15 |  |  |
| 14 |  |  | 8078.8176 | 1 |  |  |
| 16 | 8110.4927 | -42 | 8065.1733 | -17 | 8022.5010 | -44 |
| 18 | 8100.3836 | $-86^{\text {b }}$ |  |  | 8001.8092 | $74^{\text {a }}$ |
| 20 | 8088.5186 | 15 | 8032.6116 | -19 | 7979.3554 | 46 |
| 22 | 8074.8708 | -3 | 8013.6907 | -48 | 7955.1540 | -2 |
| 24 | 8059.4592 | 58 | 7993.0199 | -2 | 7929.2076 | -62 |
| 26 |  |  | 7970.5750 | $-129{ }^{\text {b }}$ |  |  |
| 28 |  |  | 7946.3943 | -52 |  |  |
| 30 |  |  | 7920.4535 | -24 |  |  |
| 32 |  |  | 7892.7557 | -20 |  |  |
| $J$ | $\mathrm{R}_{\text {ee }}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\mathrm{ee}}$ | O-C |
| 1 | 8129.9440 | 33 |  |  |  |  |
| 3 | 8133.0832 | -38 |  |  |  |  |

TABLE III. (Continued.)

|  | $7-3$ band |  |  |  |  |  |
| :--- | :---: | ---: | :---: | ---: | :---: | ---: |
| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{Q}_{\mathrm{fe}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{P}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ |
| 5 | 8134.4811 | $95^{\mathrm{b}}$ | 8118.4356 | -9 |  |  |
| 7 | 8134.0921 | -12 | 8112.7153 | -36 | 8094.0049 | $-87^{\mathrm{b}}$ |
| 9 | 8131.9513 | 2 | 8105.2453 | 29 | 8081.2181 | $179^{\mathrm{b}}$ |
| 11 | 8128.0408 | -32 | 8096.0049 | -23 | 8066.6339 | 3 |
| 13 | 8122.3719 | 9 | 8085.0134 | -1 | 8050.3192 | 39 |
| 15 | 8114.9309 | -3 | 8072.2587 | -29 | 8032.2472 | 3 |
| 17 | 8105.7164 | $-73^{\mathrm{a}}$ | 8057.7531 | 11 |  |  |
| 19 | 8094.7502 | 25 | 8041.4792 | -59 |  |  |
| 21 |  |  | 8023.4640 | 27 | 7967.5617 | 40 |
| 23 | 8067.4971 | $97^{\mathrm{b}}$ | 8003.6790 | -22 | 7942.4932 | $-124^{\mathrm{b}}$ |
| 25 | 8051.2069 | 51 | 7982.1481 | 28 |  |  |
| 27 |  |  | 7958.8547 | 2 |  |  |
| 29 |  |  | 7933.8126 | 34 |  |  |
| 33 |  |  | 7878.4563 | -22 |  |  |


|  |  | $7-4$ band |  |  |  |  |  |
| :--- | :---: | ---: | :---: | :---: | :---: | ---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | $\mathrm{Q}_{\mathrm{ef}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{P}_{\mathrm{ff}}$ | O-C |  |
| 2 |  |  | 6612.5997 | -14 |  |  |  |
| 4 | 6623.1250 | -11 | 6609.7700 | $79^{\mathrm{a}}$ | 6599.0778 | $79^{\mathrm{a}}$ |  |
| 6 | 6624.0028 | -34 | 6605.3040 | 31 | 6589.2615 | -43 |  |
| 8 | 6623.2573 | -27 | 6599.2035 | $-143^{\mathrm{b}}$ | 6577.8340 | $-94^{\mathrm{b}}$ |  |
| 10 | 6620.8915 | 50 | 6591.5126 | -3 | 6564.7979 | -63 |  |
| 12 | 6616.8841 | -6 | 6582.1829 | -36 | 6550.1511 | 14 |  |
| 14 | 6611.2658 | $120^{\mathrm{b}}$ | 6571.2378 | -13 | 6533.8874 | 58 |  |
| 16 |  |  | 6558.6574 | $-136^{\mathrm{b}}$ | 6515.9983 | -31 |  |
| 18 | 6595.1021 | 7 | 6544.4823 | -5 |  |  |  |
| 20 | 6584.5771 | -14 | 6528.6759 | 10 |  |  |  |
| 22 | 6572.4254 | 18 | 6511.2491 | 11 | 6452.7033 | -34 |  |
| 24 |  |  | 6492.1992 | -36 |  |  |  |
| 26 | 6543.2208 | 52 | 6471.5395 | -5 |  |  |  |
| 30 | 6507.4760 | 23 |  |  |  |  |  |
| 32 |  |  | 6399.8557 | 11 |  |  |  |
| 34 |  |  | 6372.7304 | 2 |  |  |  |


| $J$ | $\mathrm{R}_{\text {ee }}$ | O-C | $\mathrm{Q}_{\mathrm{fe}}$ | O-C | $\mathrm{P}_{\text {ee }}$ | O-C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 6622.0801 | 13 | 6611.3914 | 48 |  |  |
| 5 | 6623.7789 | 38 | 6607.7417 | 18 |  |  |
| 7 | 6623.8584 | $114{ }^{\text {b }}$ | 6602.4731 | 4 | 6583.7698 | 24 |
| 9 | 6622.2947 | 10 | 6595.5789 | -61 | 6571.5501 | $73^{\text {a }}$ |
| 11 | 6619.1286 | $145^{\text {b }}$ | 6587.0777 | 5 | 6557.7075 | 39 |
| 13 |  |  | 6576.9502 | 5 | 6542.2606 | $91^{\text {b }}$ |
| 15 | 6607.8440 | $-283{ }^{\text {b }}$ | 6565.2016 | -11 | 6525.1873 | -7 |
| 17 | 6599.8046 | -39 | 6551.8497 | $129{ }^{\text {b }}$ | 6506.5161 | 13 |
| 19 | 6590.1201 | 51 | 6536.8515 | -9 |  |  |
| 21 | 6578.7928 | 15 | 6520.2498 | -3 | 6464.3433 | -31 |
| 23 | 6565.8472 | $104{ }^{\text {b }}$ | 6502.0466 | $161{ }^{\text {b }}$ | 6440.8670 | $121^{\text {b }}$ |
| 25 | 6551.2540 | 32 | 6482.1973 | 30 |  |  |
| 27 | 6535.0318 | -10 |  |  | 6389.0720 | 52 |
| 29 | 6517.1835 | 10 | 6437.6691 | -63 |  |  |
| 33 | 6476.5711 | $-124^{\text {b }}$ |  |  |  |  |
| $8-5$ band |  |  |  |  |  |  |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | $\mathrm{Q}_{\text {ef }}$ | O-C | $\mathrm{P}_{\mathrm{ff}}$ | O-C |
| 2 | 6359.5396 | -40 |  |  |  |  |
| 4 | 6362.0050 | $193{ }^{\text {b }}$ | 6348.7944 | 21 |  |  |
| 6 |  |  | 6344.3339 | -50 |  |  |
| 8 | 6362.0050 | 35 | 6338.2719 | 55 |  |  |
| 10 | 6359.5829 | $97^{\text {b }}$ | 6330.5763 | 13 |  |  |

TABLE III. (Continued.)

|  |  | $8-5$ band |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | $\mathrm{R}_{\mathrm{ff}}$ | O-C | $\mathrm{Q}_{\mathrm{ef}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{P}_{\mathrm{ff}}$ | $\mathrm{O}-\mathrm{C}$ |  |
| 12 |  |  | 6321.2661 | 11 | 6289.6301 | $-76^{\mathrm{a}}$ |  |
| 14 | 6349.8431 | 35 | 6310.3464 | $95^{\mathrm{b}}$ | 6273.4552 | -20 |  |
| 16 | 6342.5326 | -1 |  |  | 6255.6658 | -18 |  |
| 18 | 6333.5957 | -24 |  |  |  |  |  |
| 20 |  |  | 6267.8552 | 68 |  |  |  |
| 22 |  |  | 6250.4544 | 16 |  |  |  |
| 24 |  |  | 6231.4410 | -11 |  |  |  |
| $J$ | $\mathrm{R}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{Q}_{\mathrm{fe}}$ | $\mathrm{O}-\mathrm{C}$ | $\mathrm{P}_{\mathrm{ee}}$ | $\mathrm{O}-\mathrm{C}$ |  |
| 3 | 6360.9681 | -15 | 6350.4150 | 10 |  |  |  |
| 5 | 6362.6131 | $91^{\mathrm{b}}$ | 6346.7785 | 49 |  |  |  |
| 7 | 6362.6131 | -38 | 6341.5182 | 27 |  |  |  |
| 9 | 6360.9681 | $-392^{\mathrm{b}}$ | 6334.6380 | -19 |  |  |  |
| 11 |  |  | 6326.1461 | -9 | 6297.1401 | $-86^{\mathrm{a}}$ |  |
| 13 | 6352.9197 | 28 | 6316.0454 | $82^{\mathrm{a}}$ | 6281.7767 | -59 |  |
| 15 | 6346.4328 | -15 |  |  | 6264.8165 | $84^{\mathrm{a}}$ |  |
| 17 | 6338.3321 | 63 |  |  |  |  |  |
| 19 |  |  | 6276.0121 | 13 |  |  |  |

${ }^{\mathrm{a}}$ With reduced weight in the least-squares fit.
${ }^{\mathrm{b}}$ Not included in the least-squares fit.
${ }^{\mathrm{c}}$ With known perturbation in the $A^{1} \Pi_{\mathrm{u}}$ state.
the energy term of the $B^{1} \Delta_{\mathrm{g}}$ state as

$$
\begin{align*}
E= & T_{\mathrm{e}}+\omega_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)-\omega_{\mathrm{e}} x_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{2} \\
& +\omega_{\mathrm{e}} y_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{3}+\omega_{\mathrm{e}} z_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{4}+\left(B_{\mathrm{e}}-\alpha_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)\right. \\
& \left.+\gamma_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{2}+\delta_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{3}\right) J(J+1) \\
& -\left(D_{\mathrm{e}}+\beta_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)+\zeta_{\mathrm{e}}\left(\mathrm{v}+\frac{1}{2}\right)^{2}\right) J^{2}(J+1)^{2} \\
& +H_{\mathrm{e}} J^{3}(J+1)^{3} \tag{1}
\end{align*}
$$

was carried out satisfactorily for up to $v=8$, as shown in Table V and in the supplementary material ${ }^{16}$ for the detailed fit. For comparison, the results of Douay et al., ${ }^{6}$ which were derived from the effective molecular constants for the individual levels up to $\mathrm{v}=5$, are also shown in Table V. Three higher order constants $\omega_{\mathrm{e}} z_{\mathrm{e}}, \delta_{\mathrm{e}}$, and $H_{\mathrm{e}}$ were obtained in this work. Some transitions with $J$ lower than 25 for $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=6$ showed a small perturbation of about $0.02 \mathrm{~cm}^{-1}$ and were not included in the global fit. We consider that this perturbation is caused by the spin-orbit interaction with the nearby $\mathrm{v}=0$ level of the $d^{3} \Pi_{g}$ state (about $170 \mathrm{~cm}^{-1}$ higher). In Sec. IV, we will show that many perturbations observed previously can be identified as interactions near level crossings between the $B^{1} \Delta_{\mathrm{g}}$ and $d^{3} \Pi_{\mathrm{g}}$ states.

In contrast to the $B^{1} \Delta_{\mathrm{g}}$ state, the molecular constants for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state are anomalous as pointed out by Douay et al. ${ }^{6}$ Effective molecular constants for the individual vibrational levels up to $\mathrm{v}=4$ of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state were obtained as shown in Table VI. In comparison with the results of Douay et al. ${ }^{6}$ for v up to 3, all the constants are in good agreement except for the $H_{v}$ constants, since we have extended the analysis to higher $J$ values. We also tried a global fit for the 17 bands of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ system using Dunham parameters, but the standard deviation of about $0.3 \mathrm{~cm}^{-1}$ for the fit was not acceptable. Therefore, a fit for the effective molecular constants listed in Table VI led to the Dunham parameters (with the same definition as the above for the $B^{1} \Delta_{\mathrm{g}}$ state) as shown in Table VII. The present result with v up to 4 showed that $\omega_{\mathrm{e}} y_{\mathrm{e}}, \gamma_{\mathrm{e}}$, and even $\omega_{\mathrm{e}} x_{\mathrm{e}}$ (with a value of $0.10(11) \mathrm{cm}^{-1}$ if fitted) were indeterminable, and we fixed them to zero. In the fit of Douay et al. with v up to 3, they obtained exact values for the four Dunham parameters from four vibrational levels up to $v=3 .{ }^{6}$ If $\omega_{\mathrm{e}} y_{\mathrm{e}}$ is fixed to zero, $\omega_{\mathrm{e}} x_{\mathrm{e}}$ becomes very small and indeterminable. So we listed also a fit for v up to 3 with $\omega_{\mathrm{e}} x_{\mathrm{e}}$ and $\omega_{\mathrm{e}} y_{\mathrm{e}}$ fixed to zero in Table VII, which gives a much better prediction (about $0.2 \mathrm{~cm}^{-1}$ shift) for the band positions involved with $\mathrm{v}=4$ of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state. The $\zeta_{\mathrm{e}}$ constant for the $(v+1 / 2)^{2}$ term of the centrifugal distortion expansions was required to account for the nonlinear dependence of $D_{v}$ on $v+1 / 2$ in Table VI. The anomalously large value of $\zeta_{\mathrm{e}}$,

TABLE IV. Effective molecular constants for the $B^{1} \Delta_{\mathrm{g}}$ state (in $\mathrm{cm}^{-1}$ ). ${ }^{\text {a }}$

| v | $T_{v}{ }^{\text {b }}$ |  | $B_{v}$ |  | $D_{v} \times 10^{6}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | This work | Douay et al. ${ }^{\text {c }}$ | This work | Douay et al. ${ }^{\text {c }}$ | This work | Douay et al. ${ }^{\text {c }}$ |
| 0 | $11859.0998(3)$ | 11859.0980 (2) | $1.4552645(13)$ | 1.4552733 (21) | 6.3186(10) | $6.3259(13)$ |
| 1 | $13243.6383(3)$ | $13243.6377(3)$ | 1.4384231 (10) | $1.4384277(24)$ | $6.3376(5)$ | $6.3420(16)$ |
| 2 | $14605.3101(4)$ | $14605.3115(4)$ | $1.4215514(16)$ | $1.4215521(28)$ | 6.3613(12) | $6.3575(24)$ |
| 3 | $15944.1788(4)$ | $15944.1799(4)$ | $1.4046440(19)$ | $1.4046420(30)$ | 6.3801(17) | $6.3671(29)$ |
| 4 | 17260.3029 (4) | 17260.3030 (12) | $1.3877050(22)$ | $1.3877210(80)$ | $6.3900(21)$ | $6.4035(85)$ |
| 5 | 18553.7491 (6) | $18553.7486(9)$ | $1.3707443(38)$ | 1.3707393 (81) | 6.4115(41) | 6.388 (14) |
| 6 | $19824.5495(6)$ |  | $1.3537985(31)$ |  | 6.4578(30) |  |
| 7 | $21072.8585(6)$ |  | 1.3367249 (33) |  | 6.4474(32) |  |
| 8 | 22 298.673(3) |  | 1.319 668(28) |  | 6.452(43) |  |

[^2]TABLE V. Dunham parameters for the $B^{1} \Delta_{\mathrm{g}}$ state $\left(\right.$ in $^{\mathrm{cm}}{ }^{-1}$ ). ${ }^{\text {a }}$

|  | This work $^{\mathrm{b}}$ | Douay et al. ${ }^{\mathrm{c}}$ |
| :--- | :---: | :---: |
| $T_{\mathrm{e}}$ | $12082.34355(54)$ | $12082.3360(40)$ |
| $\omega_{\mathrm{e}}$ | $1407.45092(77)$ | $1407.46529(134)$ |
| $\omega_{\mathrm{e}} x_{\mathrm{e}}$ | $11.47137(34)$ | $11.47937(60)$ |
| $\omega_{\mathrm{e}} y_{\mathrm{e}}$ | $0.008524(58)$ | $0.010256(73)$ |
| $\omega_{\mathrm{e}} z_{\mathrm{e}}$ | $0.0001288(34)$ |  |
| $B_{\mathrm{e}}$ | $1.4636732(16)$ | $1.4636853(34)$ |
| $\alpha_{\mathrm{e}}$ | $0.01680925(92)$ | $0.0168161(35)$ |
| $\gamma_{\mathrm{e}} \times 10^{5}$ | $-1.712(25)$ | $-1.503(72)$ |
| $\delta_{\mathrm{e}} \times 10^{7}$ | $1.81(21)$ |  |
| $D_{\mathrm{e}} \times 10^{6}$ | $6.3063(16)$ | $6.3188(19)$ |
| $\beta_{\mathrm{e}} \times 10^{8}$ | $1.778(25)$ | $1.492(113)$ |
| $H_{\mathrm{e}} \times 10^{12}$ | $-1.75(44)$ |  |

${ }^{\mathrm{a}}$ Numbers in parentheses are one standard deviation in the last digits.
${ }^{\mathrm{b}}$ Obtained directly by a global fit for 1261 transitions with a standard deviation of $0.0024 \mathrm{~cm}^{-1}$.
${ }^{c}$ Obtained from the effective constants (Ref. 6).
which has the same magnitude as the $\beta_{\mathrm{e}}$ constant (Table VII), and also the small value of $\alpha_{\mathrm{e}}$, when compared with the values of about $0.016 \mathrm{~cm}^{-1}$ for other electronic states of $\mathrm{C}_{2}$, may mean that there is a vibrational perturbation of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$ state.

## IV. DISCUSSION

## A. Interaction between the $B^{\prime 1} \Sigma_{g}{ }^{+}$and $X^{1} \Sigma_{g}{ }^{+}$ potential curves

The anharmonic constants $\omega_{\mathrm{e}} x_{\mathrm{e}}$ can be estimated using the Pekeris relation ${ }^{17}$ as $\omega_{\mathrm{e}} x_{\mathrm{e}}=B_{\mathrm{e}}\left(\omega_{\mathrm{e}} \alpha_{\mathrm{e}} /\left(6 B_{\mathrm{e}}^{2}\right)+1\right)^{2}$, which holds very nicely for all the low-lying electronic states of $\mathrm{C}_{2}$, as shown in Table VIII, except for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states, as pointed out by Douay et al. ${ }^{6}$ Therefore, the anomalously small $\omega_{\mathrm{e}} x_{\mathrm{e}} \approx 0.1(1) \mathrm{cm}^{-1}$ obtained for $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$must be an effective value, which implies that some interaction with other electronic states distorted the anharmonic potential curve back to a near-harmonic shape, at least up to $\mathrm{v}=4$. A possible mechanism for this distortion is considered below.

The vibrational term values for the $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$state are known up to $v=9 .{ }^{18}$ In our previous global analysis, ${ }^{3}$ we fitted

TABLE VII. Dunham parameters for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state (in cm ${ }^{-1}$ ). ${ }^{\mathrm{a}}$

|  | This work |  | Douay et al. ${ }^{\mathrm{b}}$ |  |
| :--- | :---: | :---: | :---: | :---: |
| $T_{\mathrm{e}}$ | $15410.33(36)$ | $15410.77(59)$ | $15409.1390^{\mathrm{c}}$ | $15410.36(55)^{\mathrm{d}}$ |
| $\omega_{\mathrm{e}}$ | $1420.36(13)$ | $1419.84(55)$ | $1424.11890^{\mathrm{c}}$ | $1420.35(24)^{\mathrm{d}}$ |
| $\omega_{\mathrm{e}} x_{\mathrm{e}}$ | 0 (fixed) | $0.10(11)$ | $2.57113^{\mathrm{c}}$ | 0 (fixed) |
| $\omega_{\mathrm{e}} y_{\mathrm{e}}$ | 0 (fixed) | 0 (fixed) | $0.46398^{\mathrm{c}}$ | 0 (fixed) |
| $B_{\mathrm{e}}$ | $1.47967(82)$ | $1.48101(30)$ |  |  |
| $\alpha_{\mathrm{e}}$ | $0.00943(29)$ | $0.01175(46)$ |  |  |
| $\gamma_{\mathrm{e}} \times 10^{5}$ | 0 (fixed) | $67(14)$ |  |  |
| $D_{\mathrm{e}} \times 10^{6}$ | $6.785(103)$ | $6.8596(136)$ |  |  |
| $\beta_{\mathrm{e}} \times 10^{6}$ | $-0.336(97)$ | $-0.1581(143)$ |  |  |
| $\zeta_{\mathrm{e}} \times 10^{6}$ | $0.1002(189)$ |  |  |  |

${ }^{\mathrm{a}}$ Numbers in parentheses are one standard deviation in the last digits.
${ }^{\mathrm{b}}$ Reference 6.
${ }^{\mathrm{c}}$ Exact values solved from the four $T_{v}$ constants of $\mathrm{v}=0-3$ in Ref. 6.
${ }^{\mathrm{d}}$ Obtained by fitting from the four $T_{v}$ constants of $\mathrm{v}=0-3$ for comparison in this work.

TABLE VIII. Calculated values for $\omega_{\mathrm{e}} x_{\mathrm{e}}$ of $\mathrm{C}_{2}$ from the Pekeris relation $\left(\mathrm{cm}^{-1}\right)$.

|  | Calculated | Experimental | Expt.-calculated |
| :--- | :---: | :---: | :---: |
| $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$ | 13.0 | $14.6(1)^{\mathrm{a}}$ | 1.6 |
| $A^{1} \Pi_{\mathrm{u}}$ | 12.1 | $12.079(2)^{\mathrm{b}}$ | 0.0 |
| $a^{3} \Pi_{\mathrm{u}}$ | 11.9 | $11.6490(4)^{\mathrm{b}}$ | -0.3 |
| $b^{3} \Sigma_{\mathrm{g}}{ }^{-}$ | 11.6 | $11.1355(4)^{\mathrm{b}}$ | -0.5 |
| $B^{1} \Delta_{\mathrm{g}}$ | 11.8 | $11.4742(4)^{\mathrm{c}}$ | -0.3 |
| $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$ | 6.0 | $0.1(1)^{\mathrm{c}}$ | -5.9 |

${ }^{\text {a }}$ From Table IX.
${ }^{\mathrm{b}}$ Taken from Ref. 3.
${ }^{\mathrm{c}}$ From the present study.
the bands up to $\mathrm{v}=6$ with $G(v)=\omega_{\mathrm{e}}\left(v+\frac{1}{2}\right)-\omega_{\mathrm{e}} x_{\mathrm{e}}\left(v+\frac{1}{2}\right)^{2}$ $+\omega_{\mathrm{e}} y_{\mathrm{e}}\left(v+\frac{1}{2}\right)^{3}+\omega_{\mathrm{e}} \mathrm{Z}_{\mathrm{e}}\left(v+\frac{1}{2}\right)^{4}+\omega_{\mathrm{e}} a_{\mathrm{e}}\left(v+\frac{1}{2}\right)^{5}$. Although the Dunham parameters obtained reproduced the transition wavenumbers accurately up to $\mathrm{v}=6$, the calculated term value of $\mathrm{v}=9$ is about $1.2 \mathrm{~cm}^{-1}$ away from the observed value. To simulate the potential curve to higher vibrational energy, we refit the term values up to $\mathrm{v}=9$ with a $G(\mathrm{v})$ expansions with just four terms, as shown in Table IX, which reproduces the vibrational term values to within $0.2 \mathrm{~cm}^{-1}$. The next expansion constant $\omega_{\mathrm{e}} a_{\mathrm{e}}$ was not determinable, so was not included in the fit.

The Dunham parameters of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states in Tables VII and IX were input to Le Roy's "RKR" program ${ }^{19}$

TABLE VI. Effective molecular constants for the individual vibrational levels of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$state (in $\mathrm{cm}^{-1}$ ). ${ }^{\text {a }}$

| v | $T_{v}{ }^{\text {b }}$ |  | $B_{v}$ |  | $D_{v} \times 10^{6}$ |  | $H_{v} \times 10^{10}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | This work | Douay et al. ${ }^{\text {c }}$ | This work | Douay et al. ${ }^{\text {c }}$ | This work | Douay et al. ${ }^{\text {c }}$ | This work | Douay et al. ${ }^{\text {c }}$ |
| 0 | $15196.5142(4)$ | $15196.5116(4)$ | 1.4752671 (32) | $1.4753124(42)$ | 6.6531(54) | 6.7810(95) | 1.260 (24) | 2.220(66) |
| 1 | 16616.9992(4) | 16616.9962(4) | 1.464779 1(29) | $1.4648230(52)$ | 6.5022(50) | 6.621(14) | $1.234(24)$ | 2.17(11) |
| 2 | 18036.5190 (4) | $18036.5144(8)$ | $1.4560621(28)$ | $1.456135(11)$ | 6.5162(49) | 6.744(35) | $1.382(22)$ | $3.38(30)$ |
| 3 | 19457.5828(4) | $19457.5801(9)^{\text {d }}$ | $1.4478405(42)$ | 1.447 863(17) | 6.9132(99) | 6.881(63) | 2.222(62) |  |
| 4 | $20878.0255(6)$ |  | $1.4366459(96)$ |  | 7.272(37) |  | 3.51(35) |  |

[^3]TABLE IX. Derived Dunham parameters and vibrational term values for the $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$state $\left(\mathrm{cm}^{-1}\right)$.

| $\omega_{\mathrm{e}}$ | $\omega_{\mathrm{e}} x_{\mathrm{e}}$ | $\omega_{\mathrm{e}} y_{\mathrm{e}}$ | $\omega_{\mathrm{e}} z_{\mathrm{e}}$ |
| :--- | :---: | :---: | :---: |
| $1856.62(36)$ | $14.61(14)$ | $0.141(22)$ | $-0.0260(11)$ |
| v | $T_{v}$ | Observed-calculated $^{\mathrm{a}}$ |  |
| 0 | 0 | 0.07 |  |
| 1 | $1827.4849(2)^{\mathrm{b}}$ | -0.17 |  |
| 2 | $3626.6835(2)^{\mathrm{b}}$ | 0.02 |  |
| 3 | $5396.6892(4)^{\mathrm{b}}$ | 0.14 |  |
| 4 | $7136.3507(6)^{\mathrm{b}}$ | 0.06 |  |
| 5 | $8844.1241(11)^{\mathrm{b}}$ | -0.10 |  |
| 6 | $10517.9659(39)^{\mathrm{b}}$ | -0.11 |  |
| 7 | $12154.9615(29)^{\mathrm{c}}$ | 0.03 |  |
| 8 | $13751.3944(38)^{\mathrm{c}}$ | 0.12 |  |
| 9 | $15302.8952(46)^{\mathrm{c}}$ | -0.06 |  |

${ }^{\text {a }}$ Observed $T_{v}$ minus calculation from the above constants.
${ }^{\mathrm{b}}$ Determined from the observation in Ref. 15.
${ }^{\text {c }}$ Determined from the observation in Ref. 18.
to calculate the RKR (Rydberg-Klein-Rees) potential curves, which are shown as the solid lines in Fig. 3. When we assume that the anharmonic expansion constants $\omega_{\mathrm{e}} x_{\mathrm{e}}$ in both the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states are $12 \mathrm{~cm}^{-1}$ as for most of the other electronic states, the dashed curves in Fig. 3 were obtained, which cross near the energy of $30000 \mathrm{~cm}^{-1}$. Since the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$ and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states have the same symmetry, the solid RKR potential curves can be regarded as the result of an avoided crossing ${ }^{20}$ of the dashed curves in Fig. 3, and the anomalous $\omega_{\mathrm{e}} x_{\mathrm{e}}$ values for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states, especially the


FIG. 3. The RKR potentials for the $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$and $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$states. The solid lines are calculated with the Dunham parameters in Tables IX and VII. They may cross around $30000 \mathrm{~cm}^{-1}$ in the case of $\omega_{\mathrm{e}} x_{\mathrm{e}}=12 \mathrm{~cm}^{-1}$ (a regular value in $\mathrm{C}_{2}$ ) and are shown as dashed lines.


FIG. 4. Rovibrational term values for the $d^{3} \Pi_{\mathrm{g}}$ state (line with open circles) and the $B^{1} \Delta_{\mathrm{g}}$ state (line with solid dots).

TABLE X. Calculated and observed perturbations around level crossings (in $\mathrm{cm}^{-1}$ ).

| $B^{1} \Delta_{\mathrm{g}}$ |  |  | $d^{3} \Pi_{\mathrm{g}}$ |  |  |  | $\Delta^{\text {c }}$ | $\left\|\left\langle\mathrm{v}_{\boldsymbol{B}} \mid \mathrm{v}_{\boldsymbol{d}}\right\rangle\right\|^{\text {d }}$ | $W^{\text {e }}$ | $\delta^{\mathrm{f}}$ | $r(\%)^{\text {g }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| v | $J$ | $E_{B}{ }^{\text {a }}$ | v | $J$ | F | $E_{d}{ }^{\text {b }}$ |  |  |  |  |  |
| 7 | 50 | 24439.58 | 0 | 50 | 2 | 24397.74 | 41.84 | 0.128 | 1.68 | 0.03 | 0.1 |
| 7 | 51 | 24572.51 | 0 | 51 | 2 | 24574.74 | -2.24 | 0.128 | 1.68 | 0.42 (0.56) | 16.2 |
| 7 | 52 | 24707.90 | 0 | 52 | 2 | 24749.79 | -41.89 | 0.128 | 1.68 | 0.03 | 0.1 |
| 7 | 54 | 24986.06 | 0 | 54 | 1 | 24932.46 | 53.60 | 0.128 | 1.68 | 0.02 | 0.1 |
| 7 | 55 | 25128.80 | 0 | 55 | 1 | 25114.51 | 14.29 | 0.128 | 1.68 | 0.08 | 0.6 |
| 7 | 56 | 25273.99 | 0 | 56 | 1 | 25304.28 | -30.29 | 0.128 | 1.68 | 0.04 | 0.2 |
| 8 | 35 | 23951.20 | 1 | 35 | 2 | 23909.61 | 41.59 | 0.221 | 2.83 | 0.08 | 0.2 |
| 8 | 36 | 24045.01 | 1 | 36 | 2 | 24030.91 | 14.10 | 0.221 | 2.83 | 0.23 (0.27) | 1.9 |
| 8 | 37 | 24141.36 | 1 | 37 | 2 | 24158.86 | -17.50 | 0.221 | 2.83 | 0.19 (0.37) | 1.3 |
| 8 | 38 | 24240.23 | 1 | 38 | 2 | 24286.71 | -46.48 | 0.221 | 2.83 | 0.07 | 0.2 |
| 8 | 40 | 24445.55 | 1 | 40 | 1 | 24420.10 | 25.45 | 0.221 | 2.83 | 0.13 | 0.6 |
| 8 | 41 | 24551.98 | 1 | 41 | 1 | 24555.16 | -3.17 | 0.221 | 2.83 | $0.84(0.74)^{\text {h }}$ | 20.7 |
| 8 | 42 | 24660.92 | 1 | 42 | 1 | 24696.04 | -35.12 | 0.221 | 2.83 | 0.10 | 0.3 |
| 9 | 6 | 23556.79 | 2 | 6 | 2 | 23534.49 | 22.29 | 0.272 | 3.48 | 0.23 (0.11) | 1.2 |
| 9 | 7 | 23575.01 | 2 | 7 | 2 | 23558.61 | 16.40 | 0.272 | 3.48 | 0.30 (0.18) | 2.1 |
| 9 | 8 | 23595.84 | 2 | 8 | 2 | 23585.24 | 10.60 | 0.272 | 3.48 | 0.46 (0.34) | 4.7 |
| 9 | 9 | 23619.27 | 2 | 9 | 2 | 23616.31 | 2.96 | 0.272 | 3.48 | 1.21 (1.13) | 27.9 |
| 9 | 10 | 23645.30 | 2 | 10 | 2 | 23649.73 | -4.43 | 0.272 | 3.48 | 0.95 (0.91) | 17.9 |
| 9 | 11 | 23673.92 | 2 | 11 | 2 | 23687.70 | -13.78 | 0.272 | 3.48 | 0.36 (0.40) | 2.9 |
| 9 | 12 | 23705.14 | 2 | 12 | 2 | 23727.88 | -22.75 | 0.272 | 3.48 | 0.22 (0.23) | 1.1 |
| 9 | 12 | 23705.14 | 2 | 12 | 1 | 23683.57 | 21.57 | 0.272 | 3.48 | 0.23 (0.18) | 1.3 |
| 9 | 13 | 23738.95 | 2 | 13 | 1 | 23724.64 | 14.31 | 0.272 | 3.48 | 0.35 (0.39) | 2.7 |
| 9 | 14 | 23775.35 | 2 | 14 | 1 | 23769.18 | 6.17 | 0.272 | 3.48 | 0.73 (0.78) | 11.2 |
| 9 | 15 | 23814.34 | 2 | 15 | 1 | 23816.93 | -2.59 | 0.272 | 3.48 | 1.31 (1.41) | 32.1 |
| 9 | 16 | 23855.92 | 2 | 16 | 1 | 23868.25 | -12.33 | 0.272 | 3.48 | 0.40 (0.49) | 3.6 |
| 9 | 17 | 23900.08 | 2 | 17 | 1 | 23922.68 | -22.61 | 0.272 | 3.48 | 0.22 (0.26) | 1.2 |
| 10 | 53 | 28309.05 | 2 | 53 | 2 | 28281.76 | 27.29 | 0.231 | 2.96 | 0.13 | 0.6 |
| 10 | 54 | 28443.79 | 2 | 54 | 2 | 28457.78 | -13.99 | 0.231 | 2.96 | 0.26 | 2.1 |
| 10 | 57 | 28862.01 | 2 | 57 | 1 | 28825.70 | 36.31 | 0.231 | 2.96 | 0.10 | 0.3 |
| 10 | 58 | 29006.05 | 2 | 58 | 1 | 29017.72 | -11.67 | 0.231 | 2.96 | 0.31 | 2.9 |
| 11 | 40 | 27904.67 | 3 | 40 | 2 | 27874.45 | 30.23 | 0.257 | 3.29 | 0.15 | 0.6 |
| 11 | 41 | 28006.88 | 3 | 41 | 2 | 28012.19 | -5.31 | 0.257 | 3.29 | 0.75 | 12.9 |
| 11 | 42 | 28111.49 | 3 | 42 | 2 | 28149.21 | -37.71 | 0.257 | 3.29 | 0.12 | 0.4 |
| 11 | 44 | 28327.90 | 3 | 44 | 1 | 28292.35 | 35.55 | 0.257 | 3.29 | 0.13 | 0.4 |
| 11 | 45 | 28439.68 | 3 | 45 | 1 | 28436.18 | 3.50 | 0.257 | 3.29 | 1.01 | 22.0 |
| 11 | 46 | 28553.83 | 3 | 46 | 1 | 28586.49 | -32.66 | 0.257 | 3.29 | 0.14 | 0.5 |
| 12 | 21 | 27555.44 | 4 | 21 | 2 | 27533.32 | 22.12 | 0.242 | 3.10 | 0.18 | 1.0 |
| 12 | 22 | 27610.22 | 4 | 22 | 2 | 27604.91 | 5.30 | 0.242 | 3.10 | $0.67(0.90)^{\text {h }}$ | 11.9 |
| 12 | 23 | 27667.46 | 4 | 23 | 2 | 27681.78 | -14.32 | 0.242 | 3.10 | 0.28 (0.49) | 2.2 |
| 12 | 26 | 27853.92 | 4 | 26 | 1 | 27841.40 | 12.51 | 0.242 | 3.10 | $0.31(0.07)^{\text {i }}$ | 2.8 |
| 12 | 27 | 27920.97 | 4 | 27 | 1 | 27926.45 | -5.48 | 0.242 | 3.10 | $0.66(0.43)^{\text {i }}$ | 11.3 |
| 12 | 28 | 27990.46 | 4 | 28 | 1 | 28016.06 | -25.60 | 0.242 | 3.10 | $0.16(0.35)^{\text {i }}$ | 0.7 |

${ }^{\text {a Calculated with the constants from this work in Table } V \text { and after subtraction of } 924.102 \mathrm{~cm}^{-1} \text {, which is the zero point energy of }}$ $X^{1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=0$ from Ref. 3.
${ }^{\mathrm{b}}$ Calculated with the constants from Ref. 13 and the addition of $613.650 \mathrm{~cm}^{-1}$, which is the energy gap between $\mathrm{v}=0$ of $a^{3} \Pi_{\mathrm{u}}$ and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$from Ref. 3.
${ }^{c}$ Energy difference $\Delta=E_{B}-E_{d}$.
${ }^{\text {d }}$ Overlap integrals calculated with Le Roy's "LEVEL" program (Ref. 22).
${ }^{\mathrm{e}}$ Off-diagonal spin-orbit interaction $W=\left|A_{d B}\left\langle\mathrm{v}_{B} \mid \mathrm{v}_{d}\right\rangle\right|$, where $\left|A_{d B}\right|=8.3(1) \mathrm{cm}^{-1}$ is an averaged value from Table XI.
${ }^{\mathrm{f}}$ Perturbation shift $\delta=\sqrt{(\Delta / 2)^{2}+W^{2}}-|\Delta / 2|$. Values in parentheses are from the observation.
${ }^{\mathrm{g}}$ Contribution (squared coefficient) ratio $r=1 /(1+\Delta / \delta)$ for $B^{1} \Delta_{\mathrm{g}}$ to $d^{3} \Pi_{\mathrm{g}}$, calculated for comparison with the intensity ratio of
the forbidden transition to the allowed ones in Table XI.
${ }^{\text {h }}$ Reassigned by this work.
${ }^{i}$ Assigned by this work.

TABLE XI. Observed forbidden and allowed transitions at level crossing of the $B^{1} \Delta_{\mathrm{g}}$ and $d^{3} \Pi_{\mathrm{g}}$ states.

| $J^{\prime}-J^{\prime \prime}$ | $B^{1} \Delta_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}$ (forbidden) |  | $d^{3} \Pi_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}($ allowed) |  |  |  | $W^{\text {b }}$ | $\left\|A_{d B}\right\|^{\text {c }}$ | $r(\%)^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{v}^{\prime}-\mathrm{v}^{\prime \prime}$ | $v\left(\mathrm{~cm}^{-1}\right)$ | $\mathrm{v}^{\prime}-\mathrm{v}^{\prime \prime}$ | $F^{\prime}-F^{\prime \prime}$ | $v\left(\mathrm{~cm}^{-1}\right)$ | $\delta^{\text {a }}$ |  |  |  |
| 41-40 | 8-0( $\mathrm{F}_{1}$ ) | 21421.436 | 1-0 | 1-1 | 21426.120 | 0.92 | 1.94 | 8.78 | $(4.6)^{\text {e }}$ |
| 41-40 | $8-1\left(\mathrm{~F}_{1}\right)$ | 19829.205 | 1-1 | 1-1 | 19833.893 | 0.74 | 1.70 | 7.70 | 15.8 |
| 9-8 | 9-1( $\mathrm{F}_{2}$ ) | 21272.042 | 2-1 | 2-2 | 21266.761 | -1.08 | 2.09 | 7.68 | 27.4 |
| 9-10 | 9-1( $\mathrm{F}_{2}$ ) | 21211.265 | 2-1 | 2-2 | 21205.888 | -1.13 | 2.15 | 7.90 | (39) ${ }^{\text {e }}$ |
| 9-8 | 9-3( $\mathrm{F}_{2}$ ) | 18108.320 | 2-3 | 2-2 | 18103.004 | -1.16 | 2.19 | 8.04 | 19.7 |
| 10-9 | $9-0\left(\mathrm{~F}_{2}\right)$ | 22884.228 | 2-0 | 2-2 | 22890.465 | 0.92 | 2.22 | 8.16 | (69) ${ }^{\text {e }}$ |
| 10-11 | $9-0\left(\mathrm{~F}_{2}\right)$ | 22816.427 | 2-0 | 2-2 | 22822.567 | 0.94 | 2.25 | 8.26 | (45) ${ }^{\text {e }}$ |
| 10-9 | 9-1( $\mathrm{F}_{2}$ ) | 21267.868 | 2-1 | 2-2 | 21273.930 | 0.92 | 2.22 | 8.16 | 13.6 |
| 10-11 | 9-1( $\mathrm{F}_{2}$ ) | 21200.641 | 2-1 | 2-2 | 21206.702 | 0.91 | 2.20 | 8.10 | 13.6 |
| 10-9 | 9-2( $\mathrm{F}_{2}$ ) | 19674.611 | 2-2 | 2-2 | 19680.722 | 0.92 | 2.22 | 8.16 | (45) ${ }^{\text {e }}$ |
| 10-9 | 9-3( $\mathrm{F}_{2}$ ) | 18104.737 | 2-3 | 2-2 | 18110.848 | 0.92 | 2.22 | 8.16 | 12.1 |
| 10-11 | 9-3( $\mathrm{F}_{2}$ ) | 18038.934 | 2-3 | 2-2 | 18045.050 | 0.93 | 2.23 | 8.21 | (4) ${ }^{\text {e }}$ |
| 14-13 | 9-1( $\mathrm{F}_{1}$ ) | 21297.543 | 2-1 | 1-1 | 21289.964 | -0.78 | 2.33 | 8.56 | (22) ${ }^{\text {e }}$ |
| 14-13 | 9-2( $\mathrm{F}_{1}$ ) | 19705.519 | 2-2 | 1-1 | 19697.852 | -0.80 | 2.36 | 8.68 | 5.5 |
| 14-13 | 9-3( $\mathrm{F}_{1}$ ) | 18136.748 | 2-3 | 1-1 | 18129.098 | -0.80 | 2.36 | 8.68 | 8.3 |
| 15-14 | 9-0( $\mathrm{F}_{1}$ ) | 22907.132 | 2-0 | 1-1 | 22912.822 | 1.37 | 2.33 | 8.56 | (58) ${ }^{\text {e }}$ |
| 15-14 | 9-2( $\mathrm{F}_{1}$ ) | 19700.759 | 2-2 | 1-1 | 19706.228 | 1.40 | 2.36 | 8.69 | 25.1 |
| 15-14 | 9-3( $\mathrm{F}_{1}$ ) | 18132.439 | 2-3 | 1-1 | 18137.933 | 1.40 | 2.36 | 8.69 | 28.0 |
| 15-16 | 9-3( $\mathrm{F}_{1}$ ) | 18040.876 | 2-3 | 1-1 | 18046.302 | 1.33 | 2.28 | 8.39 | $(12)^{\mathrm{e}}$ |


${ }^{\mathrm{b}} W=\sqrt{\delta(\delta+|\Delta|)}$, where $\Delta$ was taken from Table X .
${ }^{\mathrm{c}}\left|A_{d B}\right|=W /\left|\left\langle\mathrm{v}_{B} \mid \mathrm{v}_{d}\right\rangle\right|$, where $\left|\left\langle\mathrm{v}_{B} \mid \mathrm{v}_{d}\right\rangle\right|$ was taken from Table X.
${ }^{\mathrm{d}}$ Observed intensity ratio of the forbidden transition to the allowed ones in percentage.
${ }^{\mathrm{e}}$ Incorrect ratio due to the overlapped line intensity for the allowed or forbidden transitions.
large effect on the lower vibrational levels in $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$, are a direct result of the avoided crossing between the two states with the same ${ }^{1} \Sigma_{\mathrm{g}}{ }^{+}$symmetry. The assumption for the "original" $\omega_{\mathrm{e}} x_{\mathrm{e}}=12 \mathrm{~cm}^{-1}$ in both the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states is just a crude approximation. The two "original" potential curves may just approach each other closely instead of crossing, but the two potential curves should also be distorted mutually by the near-avoided-crossing. The distortion should also affect the higher order anharmonic constants in both the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$states and a detailed simultaneous analysis of the two states is required to determine all the parameters quantitatively. High-level ab initio calculations take this interaction into account automatically and therefore obtain reasonable estimates for the observed $\omega_{\mathrm{e}} x_{\mathrm{e}}$ values. ${ }^{7,8}$

## B. Perturbation near the level crossing of $B^{1} \Delta_{g}$ and $d^{3} \Pi_{g}$

For finding the perturbations between the $B^{1} \Delta_{\mathrm{g}}$ and $d^{3} \Pi_{\mathrm{g}}$ states, we plotted the rovibrational levels of the two states between 20000 and $30000 \mathrm{~cm}^{-1}$ in Fig. 4 and listed the term values of the levels around the level crossing in Table X. The $B^{1} \Delta_{\mathrm{g}}$ state has $\Omega=2$ and the $d^{3} \Pi_{\mathrm{g}}$ state has $\Omega=2,1$, and 0 which correspond to the $\mathrm{F}_{1}, \mathrm{~F}_{2}$, and $\mathrm{F}_{3}$ spin components, respectively, so the $\mathrm{F}_{3}$ component of $d^{3} \Pi_{\mathrm{g}}$ does not interact with the $B^{1} \Delta_{\mathrm{g}}$ state because $\Delta \Omega=2$. The term values in Table X were calculated without considering the perturbation interactions, and the perturbation shift and the intensity ratio of the forbidden to the allowed transitions borrowed from the level mixing in Table $X$ were estimated with the perturbation treatment for interaction between only two levels, ${ }^{21}$ that is,
two levels separated by $\Delta$ before interaction are shifted up and down by $\delta=\sqrt{(\Delta / 2)^{2}+W^{2}}-|\Delta / 2|$ due to the interaction $W$, and the mixed wave functions have a contribution (squared coefficients) ratio of the parent state to the perturber as $1+\Delta / \delta$. The $\mathrm{v}=0$ level of the $d^{3} \Pi_{\mathrm{g}}$ state does not cross $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=6$ and perturbs it only slightly as observed in this work. $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=0 \mathrm{~F}_{2}$ crosses $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=7$ around $J=51$, which leads to a $0.43 \mathrm{~cm}^{-1}$ shift predicted in Table X, as compared with the $0.56 \mathrm{~cm}^{-1}$ perturbation observed. ${ }^{13}$ A possible observation for the corresponding forbidden transitions at a lower wavenumber of $|\Delta|+2 \delta=3.1 \mathrm{~cm}^{-1}$ from the allowed ones is predicted with a $16 \%$ intensity of the allowed one, which was not identified due to some overlapping lines.

In this way, many perturbations observed previously ${ }^{13}$ or rechecked by this work were confirmed as shown in Table X. For $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$, the calculated perturbations around the crossing levels of $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=9$ were all confirmed and the predicted forbidden transitions from $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=9$ to $a^{3} \Pi_{u}$ were identified as shown in Table XI, among which several forbidden transitions corresponding to the allowed ones for $J=9$ of $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$ were noted previously without knowing the $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=9$ perturber. ${ }^{13}$ In Figs. 5 and 6, several identified pairs of allowed and forbidden transitions are shown. For the individual pairs of the forbidden and allowed transitions in Table XI, the interaction $W$ was determined and the off-diagonal spin-orbit constant $\left|A_{d B}\right|$ was obtained by $\mathrm{W}=\left|A_{d B}\left\langle\mathrm{v}_{B} \mid \mathrm{v}_{d}\right\rangle\right|$, where the overlap integrals were calculated with Le Roy's "LEVEL" program. ${ }^{22}$ Finally, an average value $\left|A_{d B}\right|=8.3(1) \mathrm{cm}^{-1}$ is obtained. The perturbations around
(a)

(b)


FIG. 5. Two short sections of spectra, (a) and (b), that show forbidden transitions marked with $*$ and corresponding allowed transitions for the perturbations between $\mathrm{J}=9$ and 10 of $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$ and $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=9$. The $d^{3} \Pi_{\mathrm{g}}-a^{3} \Pi_{\mathrm{u}}$ transitions are marked with $\nabla$ (assigned previously) and $\boldsymbol{\nabla}$ (assigned in this study as $P_{3}(33), P_{2}(34), P_{1}(35)$, and $P_{1}(36)$ overlapped with $P_{2}(35)$ of $v=2-3$ from the left to the right in (a)).
the level crossings of $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=2$ and $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=10$ and of $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=3$ and $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=11$ could not be confirmed since such high $J$ transitions were not assigned. As for the perturbations around the level crossings of $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=4$ and $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=12$, several perturbed transitions were found but with quite different perturbation shifts from the calculation as shown in Table X. This could be because the prediction for $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=12$ was not reliable since the observations were for transitions up to $B^{1} \Delta_{\mathrm{g}} \mathrm{v}=8$ and also because of the heavy
(a)

(b)


FIG. 6. More forbidden transitions and corresponding allowed transitions. The symbols have the same meaning as in Fig. 5. ( $\mathbf{v}$ in (a) was assigned as $\mathrm{P}_{3}$ (38) of $v=2-3$ in this study.)
perturbations in the $d^{3} \Pi_{\mathrm{g}} \mathrm{v}=4$ level, which means that the molecular constants were obtained from only a few high $J$ transitions. ${ }^{13}$

## v. CONCLUSIONS

Assignments for the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}-A^{1} \Pi_{\mathrm{u}}$ and $B^{1} \Delta_{\mathrm{g}}-A^{1} \Pi_{\mathrm{u}}$ systems of $\mathrm{C}_{2}$ have been extended to higher vibrational levels of the $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $B^{1} \Delta_{\mathrm{g}}$ states. The anharmonic constant $\omega_{\mathrm{e}} x_{\mathrm{e}}$ of nearly zero obtained for $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$indicates a distortion of the potential curve and was explained as the result of an avoided
crossing or a nearly avoided crossing with the $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$state. If an avoided crossing occurs, a nonadiabatic transition between the adiabatic $B^{\prime 1} \Sigma_{\mathrm{g}}{ }^{+}$and $X^{1} \Sigma_{\mathrm{g}}{ }^{+}$potential curves should be possible for a rapid passage through the crossing region as a Landau-Zener transition, ${ }^{23}$ which may be used to check the existence of this avoided crossing. The difficulty is that the two potential curves do not result in different products but converge to the same dissociation limit.

The higher vibrational levels of the $B^{1} \Delta_{g}$ state were found to be responsible for the many perturbations in the $d^{3} \Pi_{\mathrm{g}}$ state observed previously for the Swan band system. As proof, many forbidden transitions were identified at the level crossings. There are more level crossings between the $d^{3} \Pi_{\mathrm{g}}$ and $B^{1} \Delta_{\mathrm{g}}$ states for higher vibrational and rotational levels, which will lead to more perturbations and more forbidden transitions to be observed.

## ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI under Grant No. 15K05391. Some support was also provided by the NASA laboratory astrophysics program.
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[^1]:    ${ }^{\text {a }}$ Not included in the least-squares fit.
    ${ }^{\mathrm{b}}$ With reduced weight in the least-squares fit.

[^2]:    ${ }^{\mathrm{a}}$ Numbers in parentheses are one standard deviation in the last digits.
    ${ }^{\mathrm{b}}$ Energy term values relative to $X^{1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=0$.
    ${ }^{c}$ Reference 6.

[^3]:    ${ }^{\mathrm{a}}$ Numbers in parentheses are one standard deviation in the last digits.
    ${ }^{\mathrm{b}}$ Energy term values relative to $X^{1} \Sigma_{\mathrm{g}}{ }^{+} \mathrm{v}=0$.
    ${ }^{\mathrm{c}}$ Reference 6.
    ${ }^{\mathrm{d}}$ Corrected to $19457.8501(9) \mathrm{cm}^{-1}$ which was misprinted in Ref. 6.

